Hardness of Mastermind

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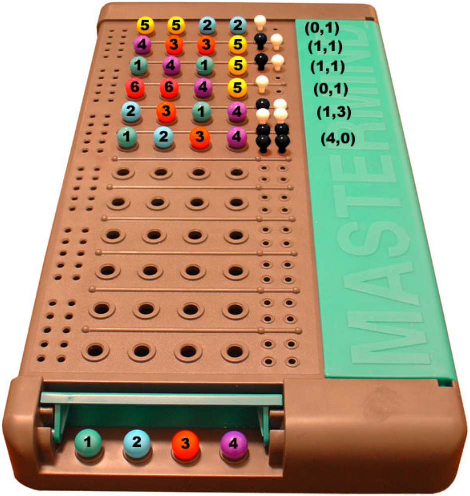
Venice - June 5$^{\text{th}}$, 2012

"Easy to learn. Easy to play. But not so easy to win."

Mastermind commercial, 1981



Mastermind is played on a board with colored pegs.
A *codemaker* chooses a secret sequence of colors,
and a *codebreaker* has to guess it in several attempts.

- After each guess, the codemaker responds with some black and white pegs.
  - Black pegs represent correct pegs in the codebreaker's guess that are also well-placed.
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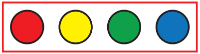
Secret code:



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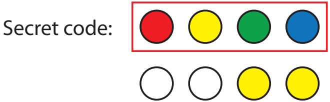

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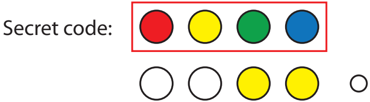

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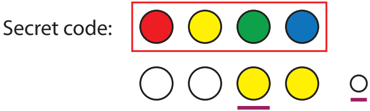

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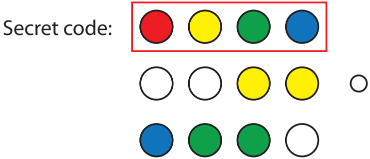

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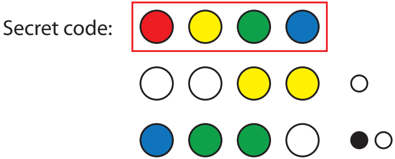

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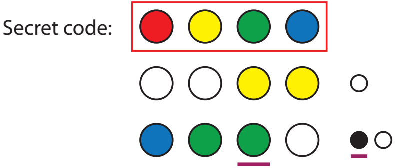

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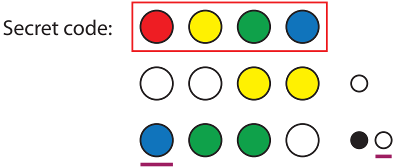

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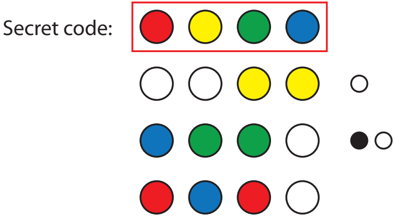

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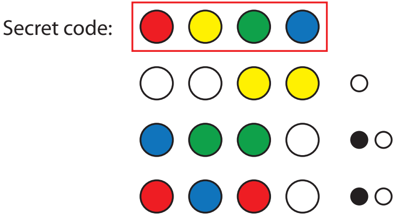

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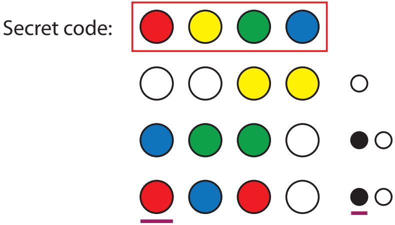

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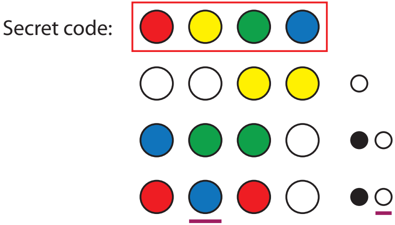

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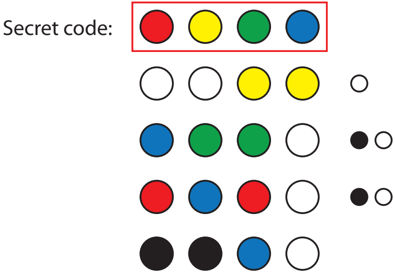

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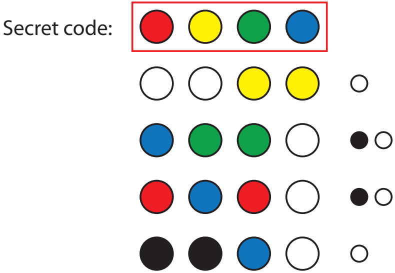

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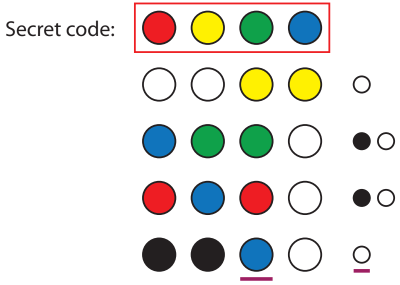

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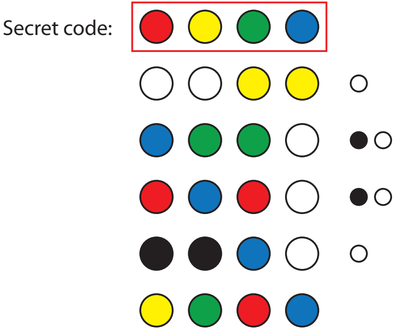

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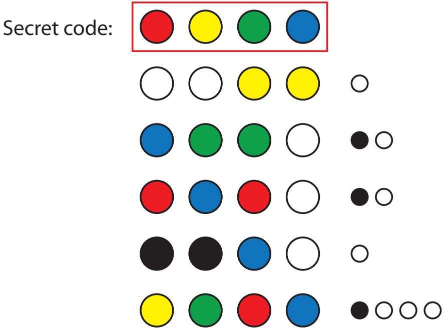

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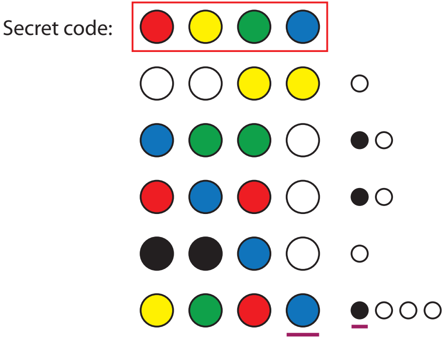

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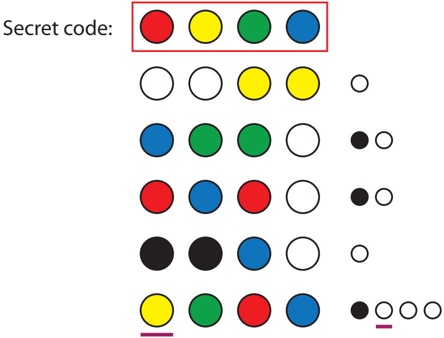

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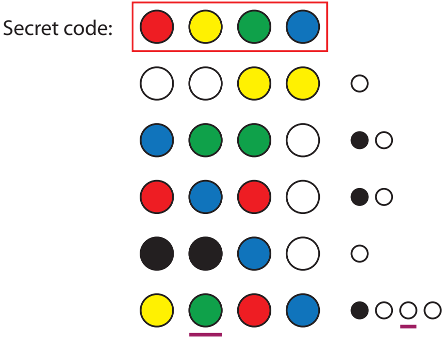

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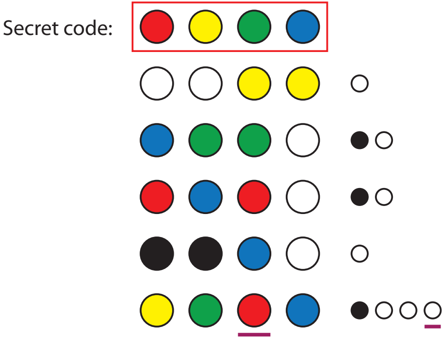

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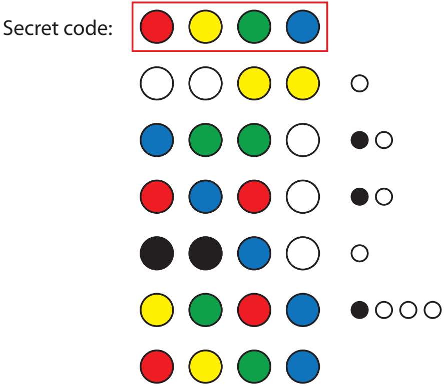

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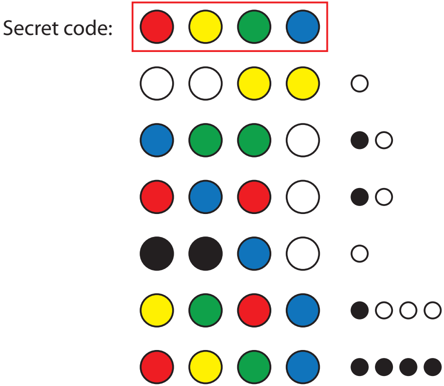
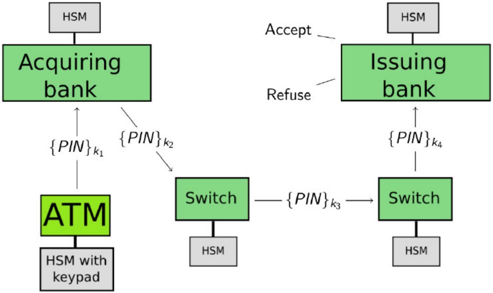


Mastermind in bank frauds

- The relevance of Mastermind in real-life security issues was pointed out in 2010 by *Focardi* and *Luccio*.
- An insider of a bank who gains access to some switch is able to issue several PIN verification API calls, eventually deducing user PINs, digit by digit.
- This kind of attack is performed exactly as an extended Mastermind game played between the insider and the bank's computers.



A feasible heuristic

- A systematic study of Mastermind was carried out by *Chvátal*, in a 1983 paper dedicated to Erdős on his 70^{th} birthday.

- Chvátal suggested a simple divide-and-conquer strategy for the codebreaker to guess the code in $2n \lceil \log c \rceil + 4n + \lceil \frac{c}{n} \rceil$ attempts. Each guess can be computed in polynomial time.

- This bound was subsequently lowered by a constant factor, with an improvement on the same basic idea.

Mastermind: a piece of cake?

- Does Chvátal's strategy trivialize the game?

Mastermind: a piece of cake?

- Does Chvátal's strategy trivialize the game?
- Not really, as long as the number of attempts is critical.
- The classic $(4, 6)$-Mastermind is solvable within 5 guesses, while Chvátal's algorithm guesses 18 times.
- Playing perfectly is still hard.

Exhaustive searches

- Another thread of heuristics was started in 1976 by *Knuth*, who devised a worst-case optimal (w.r.t. the number of guesses) greedy strategy to beat $(4,6)$-Mastermind.

- Every step of the strategy is a brute-force search among all possible guesses and all possible responses of the codemaker.
- The heuristic is based on choosing the guess that will minimize the number of eligible solutions, in the worst case.
- This is practical and optimal for $(4,6)$-Mastermind, but still infeasible and suboptimal in general.
- Several other approaches were adopted, most notably genetic algorithms, achieving different performance tradeoffs.

Satisfiability

Mastermind Satisfiability Problem (MSP)

Input: (n, c, Q), *where* Q *is any sequence of guesses and responses in* (n, c)*-Mastermind.*
Output: YES *if there exists a code that is compatible with all the queries in* Q, NO *otherwise.*

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- In 2005 *Stuckman* and *Zhang* proved that MSP is NP-complete.
- In 2009 *Goodrich* proved the same result for a variant of Mastermind where the codemaker only responds with black pegs (with an application to genetics).

Solution uniqueness and Restricted Mastermind

Problem (Stuckman–Zhang, 2005)

Can we detect MSP instances with a unique solution?

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Can we detect MSP instances with a unique solution?

- Inspired by Goodrich, we define two variants of MSP.
  - In *MSP-BLACK* the codemaker responds only with black pegs.
  - In *MSP-WHITE* the codemaker responds with $(b + w)$ white pegs whenever in MSP he would have responded (b, w). The codebreaker has to guess the code up to reordering of the pegs.

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- *c-MSP* is always played with c colors, while n is still a variable.
  - Similarly for *c-MSP-BLACK* and *c-MSP-WHITE*.

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- *c-MSP* is always played with c colors, while n is still a variable.
  - Similarly for *c-MSP-BLACK* and *c-MSP-WHITE*.
- We will determine which of these restrictions are #P-complete.

Preliminary results

Observation

In (n, c)-Mastermind restricted to white pegs, the codebreaker can guess the code after $c - 1$ attempts.

- He tries all possible colors... Although this is suboptimal.

Observation

$\#c$-*MSP-WHITE* $\in$ *FP*.

- There are only $\binom{n+c-1}{c-1} = \Theta(n^{c-1})$ possible codes to check.

Observation

$\#(c-1)$-*MSP* $\leqslant_{pars}$ $\#c$-*MSP*.

- Add the guess $ccc\cdots$ with response $(0, 0)$.
- This holds also for $\#c$-MSP-BLACK and $\#c$-MSP-WHITE.

Hardness of Mastermind

Theorem

$$\#\textit{1-IN-3-SAT} \leqslant_{pars} \begin{cases} \#\textit{MSP-WHITE} \\ \#\textit{2-MSP-BLACK} \\ \#\textit{2-MSP}. \end{cases}$$

- Hence all these variations are #P-complete.

$\#$1-IN-3-SAT $\leqslant_{pars}$ $\#$MSP-WHITE

- Let $\varphi = (x \vee \neg y \vee z) \wedge (\neg x \vee y \vee w) \wedge (y \vee \neg z \vee \neg w)$.
- Colors: $x, \bar{x}, y, \bar{y}, z, \bar{z}, w, \bar{w}, \star$.
- Code length: 4.

$\#$1-IN-3-SAT $\leqslant_{pars}$ $\#$MSP-WHITE

- Let $\varphi = (x \vee \neg y \vee z) \wedge (\neg x \vee y \vee w) \wedge (y \vee \neg z \vee \neg w)$.
- Colors: $x, \bar{x}, y, \bar{y}, z, \bar{z}, w, \bar{w}, \star$.
- Code length: 4.
- Queries:
  - $\star\star\star\star$ with score (0)

#1-IN-3-SAT $\leqslant_{pars}$ #MSP-WHITE

- Let $\varphi = (x \vee \neg y \vee z) \wedge (\neg x \vee y \vee w) \wedge (y \vee \neg z \vee \neg w)$.
- Colors: $x, \bar{x}, y, \bar{y}, z, \bar{z}, w, \bar{w}, \star$.
- Code length: 4.
- Queries:
  - $\star\star\star\star$ with score (0)
  - $x\,x\,\bar{x}\,\bar{x}$ with score (1)
  - $y\,y\,\bar{y}\,\bar{y}$ with score (1)
  - $z\,z\,\bar{z}\,\bar{z}$ with score (1)
  - $w w \bar{w} \bar{w}$ with score (1)

$\#$1-IN-3-SAT $\leqslant_{pars}$ $\#$MSP-WHITE

- Let $\varphi = (x \vee \neg y \vee z) \wedge (\neg x \vee y \vee w) \wedge (y \vee \neg z \vee \neg w)$.
- Colors: $x, \bar{x}, y, \bar{y}, z, \bar{z}, w, \bar{w}, \star$.
- Code length: 4.
- Queries:
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    - $z\ z\ \bar{z}\ \bar{z}$ with score (1)
    - $w w \bar{w} \bar{w}$ with score (1)
    - $x\ \bar{y}\ z\ \star$ with score (1)
    - $\bar{x}\ y\ w\ \star$ with score (1)
    - $y\ \bar{z}\ \bar{w}\ \star$ with score (1)

$\#$1-IN-3-SAT $\leqslant_{pars}$ $\#$2-MSP-BLACK

- Let $\varphi = (x \vee \neg y \vee z) \wedge (\neg x \vee y \vee w) \wedge (y \vee \neg z \vee \neg w)$.
- Colors: $\bullet$, $\circ$.
- Code length: 8.

$\#$1-IN-3-SAT $\leqslant_{pars}$ $\#$2-MSP-BLACK

- Let $\varphi = (x \vee \neg y \vee z) \wedge (\neg x \vee y \vee w) \wedge (y \vee \neg z \vee \neg w)$.
- Colors: $\bullet$, $\circ$.
- Code length: 8.
- Queries:

x	$\bar{x}$	y	$\bar{y}$	z	$\bar{z}$	w	$\bar{w}$	Score
$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	(4)

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x	$\bar{x}$	y	$\bar{y}$	z	$\bar{z}$	w	$\bar{w}$	Score
$\bullet$	$\bullet$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	(4)
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$\#$1-IN-3-SAT $\leqslant_{pars}$ $\#$2-MSP-BLACK

- Let $\varphi = (x \vee \neg y \vee z) \wedge (\neg x \vee y \vee w) \wedge (y \vee \neg z \vee \neg w)$.
- Colors: $\bullet$, $\circ$.
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$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	(4)

x	$\bar{x}$	y	$\bar{y}$	z	$\bar{z}$	w	$\bar{w}$	Score
$\bullet$	$\bullet$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	(4)
$\circ$	$\circ$	$\bullet$	$\bullet$	$\circ$	$\circ$	$\circ$	$\circ$	(4)
$\circ$	$\circ$	$\circ$	$\circ$	$\bullet$	$\bullet$	$\circ$	$\circ$	(4)
$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\bullet$	$\bullet$	(4)

x	$\bar{x}$	y	$\bar{y}$	z	$\bar{z}$	w	$\bar{w}$	Score
$\bullet$	$\circ$	$\circ$	$\bullet$	$\bullet$	$\circ$	$\circ$	$\circ$	(3)
$\circ$	$\bullet$	$\bullet$	$\circ$	$\circ$	$\circ$	$\bullet$	$\circ$	(3)
$\circ$	$\circ$	$\bullet$	$\circ$	$\circ$	$\bullet$	$\circ$	$\bullet$	(3)

#1-IN-3-SAT $\leqslant_{pars}$ #2-MSP

- Let $\varphi = (x \vee \neg y \vee z) \wedge (\neg x \vee y \vee w) \wedge (y \vee \neg z \vee \neg w)$.
- Colors: $\bullet$, $\circ$.
- Code length: 8.
- Queries:

x	$\bar{x}$	y	$\bar{y}$	z	$\bar{z}$	w	$\bar{w}$	Score
$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$(4,0)$

x	$\bar{x}$	y	$\bar{y}$	z	$\bar{z}$	w	$\bar{w}$	Score
$\bullet$	$\bullet$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$(4,2)$
$\circ$	$\circ$	$\bullet$	$\bullet$	$\circ$	$\circ$	$\circ$	$\circ$	$(4,2)$
$\circ$	$\circ$	$\circ$	$\circ$	$\bullet$	$\bullet$	$\circ$	$\circ$	$(4,2)$
$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\bullet$	$\bullet$	$(4,2)$

x	$\bar{x}$	y	$\bar{y}$	z	$\bar{z}$	w	$\bar{w}$	Score
$\bullet$	$\circ$	$\circ$	$\bullet$	$\bullet$	$\circ$	$\circ$	$\circ$	$(3,4)$
$\circ$	$\bullet$	$\bullet$	$\circ$	$\circ$	$\circ$	$\bullet$	$\circ$	$(3,4)$
$\circ$	$\circ$	$\bullet$	$\circ$	$\circ$	$\bullet$	$\circ$	$\bullet$	$(3,4)$

Solutions

$$(x \lor \neg y \lor z) \land (\neg x \lor y \lor w) \land (y \lor \neg z \lor \neg w)$$

x	y	z	w
T	T	T	T
T	T	T	F
T	T	F	T
T	T	F	F
T	F	F	T
F	T	T	T
F	T	T	F
F	F	T	F
F	F	F	T
F	F	F	F

x	$\overline{x}$	y	$\overline{y}$	z	$\overline{z}$	w	$\overline{w}$
●	○	●	○	●	○	●	○
●	○	●	○	●	○	○	●
●	○	●	○	○	●	●	○
●	○	●	○	○	●	○	●
●	○	○	●	○	●	●	○
○	●	●	○	●	○	●	○
○	●	●	○	●	○	○	●
○	●	○	●	●	○	○	●
○	●	○	●	○	●	●	○
○	●	○	●	○	●	○	●

Corollaries

- In a real game of Mastermind we would *know* that our queries are satisfiable. Can we use this information to compute the size of the solution space?
  - In general, is it easier to compute the number of solutions, knowing that they are at least k?

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- Then $\#\mathrm{SAT} \leqslant_T \#\mathrm{MATCH} \leqslant_{pars} \#\mathrm{MSP}$.
- The graphs with fewer than k edges are solved by hand; the others (which have at least k matchings) are mapped to #MSP.

Corollary

#MSP restricted to instances with at least k solutions is #P-hard.

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Problem (Stuckman–Zhang, 2005)

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Corollary

No, it is NP-hard under randomized Turing reductions.

Open problems

- $\#c$-MSP-WHITE $\in$ FP when c is a constant.
- $\#\sqrt[k]{n}$ -MSP-WHITE is $\#$P-complete for every $k \geqslant 1$.

Problem

What is the lowest order of growth of $c(n)$ such that $\#c(n)$-MSP-WHITE is $\#$P-complete?

Open problems

- Solving MSP is a sub-step of several heuristics.
  - But is it really necessary?

MASTERMIND

Input: (n, c, Q, k).
Output: YES *if the codebreaker has a strategy to guess the code within* k *attempts, given the set of queries* Q*.* NO *otherwise.*

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- MASTERMIND $\in$ PSPACE, due to Chvátal's strategy.

Problem

Is MASTERMIND PSPACE-complete?